Sanjay Ghodawat University, Kolhapur 2019-20

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Year and Program:2019-20 S.Y.B.Tech | **School of Technology** | **Department: Aeronautical Engineering**

Course Code: AET 227 | **Course Title: Differential Calculus and Transforms** | **Semester – III**

Day and Date: Monday 18/11/19 | **End Semester Examination (ESE)** | **Time:** 10.30am to 1.30pm | **Max Marks: 100**

Instructions:
1) All questions are compulsory.
2) Assume suitable data wherever necessary.
3) Figures to the right indicate full marks.

		Marks	Bloom's Level	CO
Q 1				
a)	Solve $(D^2-1)y = x\sinh x$.	07	L_3	CO1
	OR			
a)	Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$.	07	L_3	CO1
b)	Solve $x^2p^2 + y^2q^2 = z^2$.	08	L_3	CO2
	OR			
b)	Solve $y^2q^2 = z(z - px)$	08	L_3	CO2
Q.2				
a)	Find the Fourier series of $f(x) = \begin{cases} \pi x & ; 0 \le x \le 1, \\ \pi(2-x) & ; 1 \le x \le 2. \end{cases}$	07	L_2	CO3
	OR			
a)	Find the Fourier series of $f(x) = e^{-ax}$ in the interval $[0, 2\pi]$.	07	L_2	CO3
b)	Find Laplace transform of $f(t)$, where $f(t) = \begin{cases} \sin wt \; ; o < t < \frac{\pi}{w}, \\ 0 \quad ; \frac{\pi}{w} < t < \frac{2\pi}{w}, \end{cases}$ and $f\left(t + \frac{2\pi}{w}\right) = f(t)$.	08	L_3	CO4
	OR			

ESE

b) Find Laplace transform of $\left(\frac{\sin 2t \cos t}{t}\right)$. 08 L_2 CO4

Q.3 **Solve any Two of the following.**

a) Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. 08 L_3 CO1

b) Solve $\left(x^2+y^2\right)(p^2+q^2)=1$. 08 L_3 CO2

c) Obtain Fourier series of $f(x)=\begin{cases}-\pi & ;-\pi\le x\le 0,\\ x & ;\ 0\le x\le \pi,\end{cases}$ 08 L_4 CO3

and, hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots$.

d) Using Laplace transform evaluate $\int_0^\infty t^2e^{-3t}\sin t\,dt$. 08 L_4 CO4

Q.4 **Solve any Three of the following.**

a) Find $L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$. 06 L_1 CO4

b) Find $L^{-1}\left\{\frac{1}{(s-2)(s+2)^2}\right\}$ by using convolution theorem. 06 L_1 CO4

c) Find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$. 06 L_2 CO4

d) Using Laplace transform find the solution of initial value problem 06 L_4 CO4

$\frac{d^2y}{dt^2}+\frac{dy}{dt}=2$ with $y(0)=3,\ y'(0)=1$.

Q.5 **Solve any Three of the following.**

a) Find the directional derivative of $\phi=x^2y+y^2z+z^2x$ at $P(2,2,2)$ in the direction of the normal to the surface $4x^2y+2z^2=2$ at $Q(2,-1,3)$. 06 L_2 CO5

b) If $\phi=x^3+y^3+z^3-3xyz$ find i) div $\overline{f}$ ii) curl $\overline{f}$, where $\overline{f}=\nabla\phi$. 06 L_1 CO5

ESE

c) Find the constants a, b, c if the directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at $(1, 1, 1)$ has maximum magnitude 15 in the direction parallel to the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. — 06 — L_2 — CO5

d) Show that the vector field represented by $\overline{f} = (z^2 + 2x + 3y)i + (3x + 2y + z)j + (y + 2zx)k$ is irrotational but not solenoidal. Also obtain a scalar unction ϕ such that grad $\phi = \overline{f}$. — 06 — L_2 — CO5

Q.6 **Solve any Three of the following.**

a) Find $L^{-1}\left\{\frac{6s-4}{(s+1)(2s+1)^2}\right\}$. — 06 — L_2 — CO4

b) Using Laplace transform find the solution of initial value problem $y''(t) + y(t) = t$ given that $y(0) = 1, y(\pi) = 0$. — 06 — L_4 — CO4

c) Given that $\phi = 2x^3y^2z^4$ then show that $\nabla \cdot \nabla\phi = \nabla^2\phi$, where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplace operator. — 06 — L_6 — CO5

d) Prove that $\overline{f} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is solenoidal and determine the constants a, b, c if $\overline{f}$ irrotational. — 06 — L_2 — CO5

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